On the intermittent energy transfer at viscous scales in turbulent flows.

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Abstract

In this letter we present numerical and experimental results on the scaling properties of velocity turbulent fields in the range of scales where viscous effects are acting.

A generalized version of Extended Self Similarity capable of describing scaling laws of the velocity structure functions down to the smallest resolvable scales is introduced. Our findings suggest the absence of any sharp viscous cutoff in the intermittent transfer of energy.

The word anomalous scaling (AS) usually refers to scaling laws in a physical system which cannot be deduced from naive dimensional arguments. It is always a challenging problem in physics to understand the origin of anomalous scaling and to formulate a predictive theoretical framework to compute the anomalous scaling exponents.

Among the many physical systems showing anomalous scaling, fully developed three dimensional turbulence (FDT) has been widely investigated in the last few years (see [1] for a recent overview of the experimental and theoretical state of the art). According to Kolmogorov 1941 theory [2] the small scale statistical properties of FDT obey the relation:

$$< |\delta v(r)|^p >\sim A_p(\frac{r}{L})^{p/3} v_0^p \sim A_p \varepsilon^{p/3} r^{p/3} \tag{1}$$

where $\delta v(r) = v(x+r) - v(x)$ is the difference of velocity at scale r, v_0 is the *rms* velocity at the integral scale L, ε is the mean energy dissipation and the A_p's are dimensionless constants. Eq. (1) is not satisfied both in real experiments and numerical simulation. Indeed, one has to replace it by anomalous (also known as intermittent) scaling:

$$< |\delta v(r)|^p >\sim B_p(\frac{r}{L})^{\zeta(p)} v_0^p, \tag{2}$$

where $\zeta(p)$ is now a non-linear function of its argument.

At variance with expression (1), the scaling (2) is anomalous in the sense that it cannot be deduced by naive dimensional counting. In order to get a more precise measurements of the $\zeta(p)$ exponents and to highlight the anomalous scaling, it has been proposed in [3] [4] to look at the self-scaling properties of the velocity structure functions, namely:

$$< |\delta v(r)|^p >\sim< |\delta v(r)|^q >^{\beta(p,q)} . \tag{3}$$

This new way of looking at the scaling properties has been tested in many different experimental and numerical instances [5]. In all cases, when small-scales homogeneity and isotropy were satisfied, a dramatic improvement in the width of the scaling region was observed. This almost universal property of turbulent flows was then called Extended-Self-Similarity (ESS). ESS must interpreted as the signature of some non trivial universal physics happening at the transition between the inertial and viscous scale. It tells us that, by using the appropriate functional form, scaling is present also at scales where in principle viscous effects should already be important.

The aim of this letter is twofold. First we discuss in more details some cases where ESS does not work (shear flows and boundary layers). Second, we present a generalized version of ESS (G-ESS) which turns out to be much more universal and allows us to draw a concrete theoretical framework of the energy cascade down to the smallest resolvable scale, i.e. in a region where no anomalous scaling was supposed to be detected.

The physical outcome of our findings is that whatever is the mechanism responsible for anomalous scaling in FDT, this mechanism is acting also at extremely small scales and within experimental errors no evidence of a cutoff (due to dissipation) is observed.

We have performed a direct numerical simulation of 3 dimensional Navier-Stokes eqs. for a Kolmogorov flow (see [6] for technical details). The flow is forced such that the stationary solution has a non-zero spatial dependent mean velocity $< \vec{v}(\vec{x}) >= \hat{x} sin(\frac{8\pi}{L} z)$, where $\hat{x}$ is the versor in the direction x, and L is the integral scale.

In figures 1a and 1b we show the standard ESS analysis by plotting $< |\delta v(r)|^6 >$ versus $< |\delta v(r)|^3 >$ for two specific levels z_a and z_b, where z_a was chosen at minimum shear and z_b at maximum shear (in this case $< \cdots >$ must be

interpreted as averages over time integration at fixed z-level). The Re_λ number of the simulation was 40 and no scaling laws were present if examined as a function of the physical scale r. Nevertheless, it is clear from figure 1a that ESS is observed for the case of minimum shear and it is not observed for the case of maximum shear (figure 1b). Violations of ESS have already been reported in other cases where strong shear effects were argued to be relevant [7] [4].

In order to understand this phenomenological transition between strong shear and weak shear flows let us recall two recent results obtained by using ESS concepts.

The first is a generalized form of the Kolmogorov Refined Similarity Hypothesis (KRS) [8]:

$$< |\delta v(r)|^p > \sim < \epsilon(r)^{p/3} >< |\delta v(r)|^3 >^{p/3}, \tag{4}$$

where $< \epsilon(r)^{p/3} >$ is the dissipation energy averaged over a box of radius r. The second result follows from the moment hierarchy recently proposed in [9] and rewritten in terms of structure functions:

$$\frac{< |\delta v(r)|^{p+1} >}{< |\delta v(r)|^p >} = A_\infty(r)^{1-\gamma} \left(\frac{< |\delta v(r)|^p >}{< |\delta v(r)|^{p-1} >} \right)^\gamma . \tag{5}$$

where

$$A_\infty(r)^{1-\gamma} = A_p \left(\frac{< |\delta v(r)|^6 >}{< |\delta v(r)|^3 >^{1+\gamma^3}} \right)^{\frac{(1-\gamma)}{3(1-\gamma^3)}} , \tag{6}$$

and $\gamma^3 = 2/3$. Equation (4) was proposed in [5] and checked systematically in [10], while equations (5) has been discussed in [11]. Let us remark that (5) can be obtained from the original She-Leveque hierarchy on the energy dissipation plus the generalized KRS (4).

The novel point of (4) and (5) is that they holds also for values of r where ESS is no longer satisfied. In order to highlight the previous comment, let us

consider again the Kolmogorov flow previously observed. In figure 2a and 2b we show the result of the scaling obtained by using (4) at the correspondent z-levels of fig 1a and 1b and for $p = 6$. As one can see, the generalized KRS is well satisfied in both cases although for the z_b ESS is not observed.

Figures 2, and the results obtained in [5, 11], suggest that the concept of ESS could be generalized in such a way to take into account the scaling relations (4) and (5) properly.

For this purpose we introduce the dimensionless structure functions:

$$G_p(r) = \frac{< |\delta v(r)|^p >}{< |\delta v(r)|^3 >^{p/3}}. \tag{7}$$

According to Kolmogorov theory $G_p(r)$ should be a constant both in the inertial range and in the dissipative range, although the two constant are not necessarily thought to be the same. Because of the presence of anomalous scaling, $G_p(r)$ are no longer constants in the inertial range and, by using (4) we have:

$$G_p(r) =< \epsilon(r)^{p/3} > \tag{8}$$

Following the results shown in figs 2 and in [5, 11] equation (8) is valid for all scales and also in the case where ESS is not verified. Therefore, it seems reasonable to study the self scaling properties of $G_p(r)$ or, equivalently, the self-scaling properties of the energy dissipation averaged over a ball of size r:

$$G_p(r) = G_q(r)^{\rho(p,q)}, \tag{9}$$

where we have by definition:

$$\rho(p, q) = \frac{\zeta(p) - p/3\zeta(3)}{\zeta(q) - q/3\zeta(3)}, \tag{10}$$

$\rho(p, q)$ is given by the ratio between deviation from the K41 scaling. It will play an essential role in our understanding of energy cascade. Indeed, it is

easy to realise that it is the only quantity that can stay constant along all the cascade process: from the integral to the sub-viscous scales. It is reasonable to imagine that the velocity field becomes laminar in the sub-viscous range, $< |\delta v(r)|^p >\sim r^p$, still preserving some intermittent degree parametrized by the ratio between correction to K41 theory.

Let us give a theoretical argument behind relation (9). All of the proposed cascade-model of turbulence based on infinitely divisible distribution [1] lead to a set of scaling exponents with the following form:

$$\zeta(p) = ap + bf(p), \tag{11}$$

where a, b are model-dependent constants and f is the (model-dependent) non-linear function giving the intermittent corrections to K41. In the Log-Poisson model [9], for example, we have $f(p) = 1 - (\frac{2}{3})^{p/3}$. ESS is a statement concerning the ratio $\zeta(p)/\zeta(3)$ as a function of the analysed range of scales. Based on the expression (11) a possible interpretation is that the free parameters a and b acquires a weak scale-dependency near the viscous cutoff such that the ratio a/b stays constant. The ESS violation would mean that for very small r (comparable with the viscous cutoff) and/or in presence of a strong shear the r-dependency of a and b is no longer the same. However, if we make the (very strong) assumption that the non linear transfer of energy is always working, regardless on how a and b depend on r, then we should expect that the non linear (anomalous) contribution to $\zeta(p)$ is independent of r, i.e. $f(p)$ in expression (11) must not depend on the scale even near and inside the viscous range. It follows that the ratio between anomalies (10), using (11)

$$\rho(p, q) = \frac{f(p) - \frac{p}{3}f(3)}{f(q) - \frac{q}{3}f(3)},$$

must be a r-independent quantity. Thus the scaling relation (9) should always be observed.

In order to support this claim we have plotted in figures 3 and 4 the scaling of $G_6(r)$ versus $G_5(r)$ for many different experimental set up[3, 14, 15] done at different Reynolds number and for some direct numerical simulation with and without large scale shear. As one can see the straight line behaviour is very well supported. Within experimental errors (of the order of 3%) no deviations from the scaling régime are detected. Similar results are obtained, using different $G_p(r)$ and $G_q(r)$. In a more detailed version of this study [12] we will discuss how it is possible to reconcile the idea of multiplicative cascades with this continuous energy transfer from the inertial range to the viscous range and we will present numerical evidences that this new scaling behaviour is in disagreement with the idea of a statistical dependent viscous cutoff as predicted in all the standard multiplicative multifractal models [13]

Our results may have theoretical and applied implication. For instance, the presence of intermittent fluctuations at all scales might cast serious doubts on the validity of renormalized perturbative expansion of the NS equations which are usually based on perturbative expansion around the linearized equations.

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FIGURE CAPTIONS:

FIGURE 1a: Log-Log plot showing ESS scaling for the longitudinal structure functions, $|\delta v(r)|^6$ versus $|\delta v(r)|^3$. Data are taken from a direct numerical simulation of a shear flow at $Re_\lambda = 40$. Each point corresponds to a space separation of a Kolmogorov scale. The computation of structure functions is performed at points where the shear is minimum. The dashed line is the best fit for the slope in the scaling region.

FIGURE 1b: The same of figure 1a but for points where the shear is maximum. At variance with the previous case ESS is not observed.

FIGURE 2a: Check of equation (4) for $p = 6$ at points of minimum shear (in Log-Log scale). Energy dissipation has been computed by using the 1-dimensional surrogate, in order to compare this result with laboratory experiments [5]. The dashed line is the best fit.

FIGURE 2b: The same of figure 2a but for points of maximum shear. Although in this case ESS is not observed (see fig. 1b), generalized-ESS works within 3%.

FIGURE 3:Log-log plot of $G_6(r)$ versus $G_5(r)$ for different laboratory and numerical experiments. Data taken in a wake behind a cylinder, where standard ESS was not observed [3], (Crosses). Data taken from the region with log-profile of a boundary layer (courtesy of G. Ruiz Chavarria) where standard ESS was not observed (Circles). Data taken from a direct numerical simulation of thermal convection [14] where standard ESS was observed (Squares). Data from a direct numerical simulation of a channel flow where standard ESS was not observed [15] (Triangles).

FIGURE 4: The same as in figure 3 but for the direct numerical simulation of the shear flow.

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